Fluid Injection Through One Side of a Long Vertical Channel

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A recent article (Wang, 1973) demonstrated that exact solutions to the Navier-Stokes equations could be obtained near a three-dimensional stagnation point caused by the impinging of a circular jet on a moving plate. The present paper studies a different three-dimensional problem, that is, fluid injection through one side of a long vertical channel. A set of nonlinear ordinary differential equations results and these can be integrated numerically.

ANALYSIS

The physical model is shown in Figure 1. A fluid is injected through the porous plate at $y = d$ with uniform velocity V. The plate at $y = 0$ is impermeable. Due to gravity, which acts in the z direction, the fluid eventually flows out through the sides and the bottom of the plates. We assume $L_2 >> L_1 >> d$ such that edge effects can be ignored and the isobars are parallel to the z axis.

Let u, v, w be the velocity components in the directions x, y, z, respectively. Utilizing the symmetry of the problem, we substitute

$$u = \frac{V}{d} x f'(\eta) \tag{1}$$

$$v = -Vf(\eta) \tag{2}$$

$$w = \frac{d^2 g}{\nu} h(\eta) \tag{3}$$

$$p = \frac{-\rho}{2}\left[\frac{AV^2}{d^2}x^2 + v^2 - 2\nu\, v_y\right] \tag{4}$$

$$\eta = y/d \tag{5}$$

into the Navier-Stokes. The following set of ordinary differential equations is obtained:

$$f''' - R[(f')^2 - ff''] + RA = 0 \tag{6}$$

$$h'' + Rfh' + 1 = 0 \tag{7}$$

Here $R = Vd/\nu$ is the cross flow Reynolds number and A is a constant to be determined. Differentiating Equation (6) once we have

$$f'''' - R(f'f'' - ff''') = 0 \tag{8}$$

The boundary conditions are

$$f(0) = 0 \quad f'(0) = 0 \quad f(1) = 1 \quad f'(1) = 0$$

$$h(0) = 0 \quad h(1) = 0 \tag{9}$$

The constant A is obtained from Equation (6)

$$A = -\frac{1}{R} f''(0) \tag{10}$$

If R were small, we can expand $f(\eta)$ and $h(\eta)$ in power series in R. Equations (7) to (9) to yield

$$f(\eta) = f_0 + Rf_1 + R^2 f_2 + \ldots \tag{11}$$

$$h(\eta) = h_0 + Rh_1 + R^2 h_2 + \ldots \tag{12}$$

where

$$f_0 = 2\xi^3 - 3\xi^2 + 1 \tag{13}$$

$$f_1 = \frac{-2}{35}\xi^7 + \frac{1}{5}\xi^6 - \frac{3}{10}\xi^5 + \frac{1}{2}\xi^4 - \frac{43}{70}\xi^3 + \frac{19}{70}\xi^2 \tag{14}$$

$$f_2 = \frac{-4}{5775}\xi^{11} + \frac{2}{525}\xi^{10} - \frac{1}{210}\xi^9 - \frac{11}{560}\xi^8 + \frac{191}{2450}\xi^7 - \frac{68}{525}\xi^6 + \frac{27}{175}\xi^5 - \frac{43}{280}\xi^4 + \frac{32189}{323400}\xi^3 - \frac{17719}{646800}\xi^2 \tag{15}$$

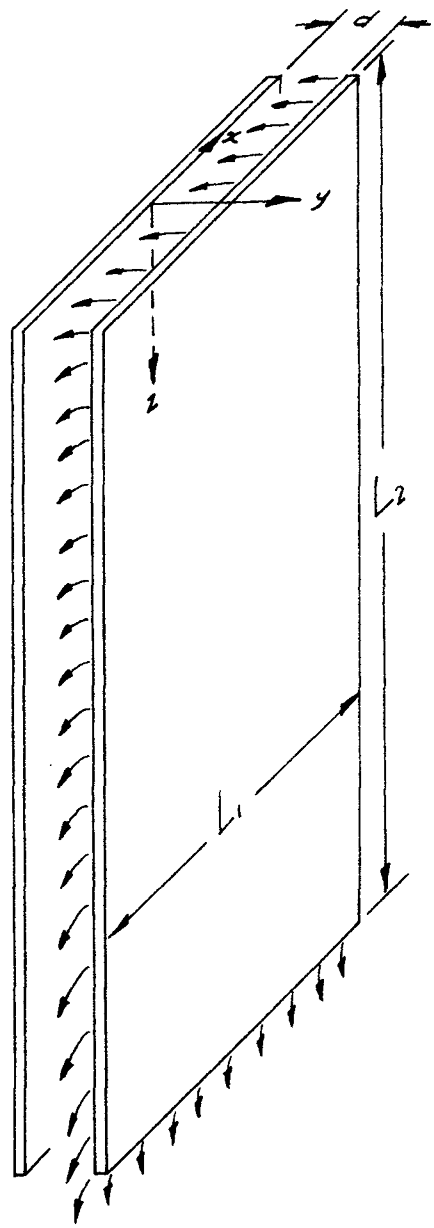

Fig. 1. The physical model.

TABLE 1. COMPARISON OF APPROXIMATE VALUES TO EXACT VALUES

R	$f''(0)$ Exact	$f''(0)$ Series Equation (11)	$f'''(0)$ Exact	$f'''(0)$ Series Equation (11)	$h'(0)$ Exact	$h'(0)$ Series Equation (12)
0.00000	6.00000	6.00000	−12.00000	−12.00000	0.50000	0.50000
0.21198	6.09679	6.09680	−12.49299	−12.49302	0.49820	0.49820
1.04092	6.47271	6.47330	−14.46473	−14.46786	0.49050	0.49044
3.96465	7.74724	7.77542	−21.85485	−22.02949	0.45794	0.45417

TABLE 2. HEAT TRANSFER ON WALLS

P \ R	$\theta'(0)$ 0.00000	$\theta'(0)$ 1.04092	$\theta'(0)$ 12.19261	$\theta'(0)$ 25.60532	$\theta'(1)$ 0.00000	$\theta'(1)$ 1.04092	$\theta'(1)$ 12.19261	$\theta'(1)$ 25.60532
0.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.20000	1.02998	1.03081	1.03614	1.03859	0.93197	0.93118	0.92676	0.92514
1.00000	1.14901	1.15326	1.18084	1.19376	0.69691	0.69373	0.67595	0.66947
10.00000	2.17229	2.20641	2.43993	2.56284	0.01464	0.01369	0.00922	0.00789

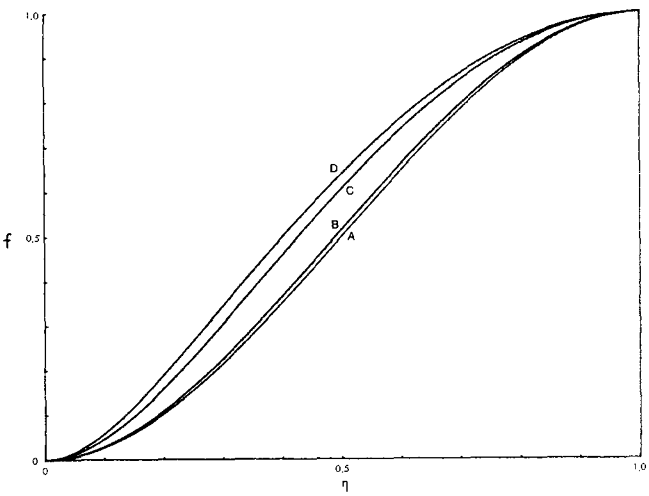

Fig. 2. Normal velocity distribution Curve A: $R = 0$; Curve B: $R = 1.04092$; Curve C: $R = 12.19261$; Curve D: $R = 25.60532$.

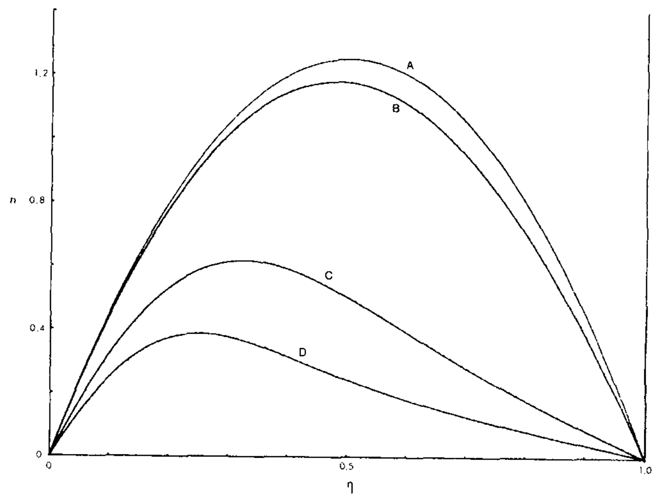

Fig. 3. Velocity profiles in the (axial) z direction Curve A: $R = 0$; Curve B: $R = 1.04092$; Curve C: $R = 12.19261$; Curve D: $R = 25.60532$.

$$h_0 = \frac{1}{2}\xi - \frac{1}{2}\xi^2 \tag{16}$$

$$h_1 = \frac{-1}{15}\xi^6 + \frac{1}{5}\xi^5 - \frac{1}{8}\xi^4 - \frac{1}{6}\xi^3 + \frac{1}{4}\xi^2 - \frac{11}{120}\xi \tag{17}$$

$$h_2 = \frac{-13}{1575}\xi^{10} + \frac{13}{315}\xi^9 - \frac{9}{140}\xi^8 - \frac{11}{840}\xi^7 + \frac{611}{4200}\xi^6 - \frac{29}{210}\xi^5 - \frac{5}{672}\xi^4 + \frac{1}{12}\xi^3 - \frac{11}{240}\xi^2 + \frac{349}{50400}\xi \tag{18}$$

and

$$\xi \equiv 1 - \eta \tag{19}$$

NUMERICAL INTEGRATION

Equation (8) was studied by Terrill (1964), albeit with different boundary conditions. The two-point boundary value problem could be transformed into an initial value problem by Terrill's scheme. We set

$$f = \alpha F(\lambda) \qquad \lambda = \alpha R \eta \tag{20}$$

Equations (8) and (9) become

$$F'''' - F'F'' + FF''' = 0 \tag{21}$$

$$F(0) = F'(0) = F'(\alpha R) = 0 \quad F(\alpha R) = \frac{1}{\alpha} \tag{22}$$

We guess $F''(0)$ and $F'''(0)$ and integrate Equation (21) by the Runge Kutta algorithm until $F' = 0$ again, say at $\lambda = \lambda^*$. Setting $\alpha R = \lambda^*$ and $F(\lambda^*) = 1/\alpha$ we obtain the constants α and R. Thus $f(\eta)$ is found.

Once $f(\eta)$ is known, we can integrate Equation (7) with a shooting method. An arbitrary $h'(0)$ is used to start the Runge Kutta algorithm. If $h(1)$ were not zero at the other end we adjust $h'(0)$ and integrate again. Due to the linearity of Equation (7) only two integrations are required to obtain the correct starting value. The results are plotted in Figures 2 and 3 for several Reynolds numbers. As the cross flow Reynolds number R is increased, the normal velocity $f(\eta)$ changes little while the axial velocity, which is due to gravity, is greatly decreased. Also, due to injection, the location of maximum downward velocity shifts towards the impermeable wall at $\eta = 0$. Table 1 shows a comparison of the values from numerical integration and those from the series expansion. The series solution is fairly accurate for low Reynold's numbers.

HEAT TRANSFER

The present problem would be important in transpiration cooling or gaseous diffusion processes. Let the temperature of the plates at $y = 0$ and $y = d$ be T_0 and T_1, respectively. We want to calculate the heat transfer.

Substituting

$$T = T_0 + (T_1 - T_0)\theta(\eta) \tag{23}$$

into the energy equation we obtain

$$\theta'' + Pf\theta' = 0 \tag{24}$$

$$\theta(0) = 0 \quad \theta(1) = 1 \tag{25}$$

where P is the Peclet number $(\rho g C_p V d/k)$. Equation (24) is integrated by a similar shooting method. The heat transfer rate is

$$Q(\eta) = -\frac{k(T_1 - T_0)}{d}\,\theta'(\eta) \tag{26}$$

The values $\theta'(\eta)$ for $\eta = 0$ (impermeable plate) and $\eta = 1$ (porous plate) are given in Table 2 for various Reynolds numbers and Peclet numbers. For high Peclet numbers the heat transfer on the impermeable plate is considerably higher than that of the porous plate.

NOTATION

A = constant which determines pressure distribution
C_p = specific heat at constant pressure
d = distance between plates
f = normal velocity distribution
F = normal velocity distribution
g = gravity acceleration
h = axial velocity distribution
k = thermal conductivity
L_1 = width of plates
L_2 = length of plates
P = Peclet number
Q = heat transfer per area per time
R = cross flow Reynolds number
T = temperature
T_0 = temperature at $\eta = 0$
T_1 = temperature at $\eta = 1$
u = velocity component in x direction
v = velocity component in y direction
V = velocity of injection
w = velocity component in z direction
x = cartesian coordinate
y = cartesian coordinate
z = cartesian coordinate

Greek Letters

α = proportionality constant
η = nondimensional normal distance
θ = temperature distribution
λ = $\alpha R \eta$
λ^* = nonzero root of $F'(\lambda) = 0$
ν = kinematic viscosity
ξ = $1 - \eta$
ρ = density

LITERATURE CITED

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Correlations for Solid Friction Factors in Vertical and Horizontal Pneumatic Conveyings

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Unlike friction factors for gases which are very well characterized with a universal measurement of turbulence, the Reynolds number, attempts to correlate solid friction factors are hampered by a lack of an unique dimensionless group. The complications stem from the fact that solid particles are usually different in size, shape, and surface roughness; that different size particles travel at different velocities and collide among one another as well as with pipe walls; that static electricity may be important, especially for small particles, but is difficult to quantify; and that the particles may not exist as a uniform suspension especially in horizontal conveying where gravitational forces tend to create a radial distribution of particle concentration.

This note presents new correlations for calculating the solid friction factors in both horizontal and vertical pneumatic conveying lines. The correlations proposed here can be used in conjunction with the modified terminal velocity equations suggested earlier (Yang, 1973a, 1973b) to calculate solid particle velocities in pneumatic conveying lines.

CORRELATIONS FOR SOLID FRICTION FACTORS

Information on the flow of fluids through beds of granular solids has been utilized to shed light on the functional dependence between different variables. Blake (1922) first obtained the correct dimensionless groups for correlating the pressure drop data in a packed bed. The suggested dimensionless groups are

$$\frac{\Delta P \cdot g_c}{2\,\rho_f\,U_0^2}\cdot\frac{d_p}{L}\cdot\frac{\epsilon^3}{(1-\epsilon)} \quad \text{and} \quad \frac{d_p\,\rho_f\,U_0}{\mu(1-\epsilon)} \tag{1}$$

The first of these groups is a modified friction factor and the second is a modified Reynolds number. Later Ergun